New Methods For Predictability Analysis

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LONG-TERM GOAL

Our long-term goal is to improve atmospheric and oceanic deterministic forecast capability by maximizing the accuracy of the initial state estimate from which a forecast is made while minimizing observational and computational costs required for obtaining the initial state.

OBJECTIVES

Our present objective is to increase fundamental understanding of error dynamics and develop practical methods for computing error statistics so that optimal state estimation algorithms can be implemented. We intend to accomplish this by reducing the dimension of the error system so that error statistics can be practically obtained using forecast products. In addition from knowledge of the statistical structure of the time dependent error field we intend to determine the areas in space and time where observational resources can be most effectively applied.

APPROACH

Small error dynamics is governed by linear equations but because these equations are non-normal and time dependent theoretical tools for analyzing error growth have only recently been developed. Our approach is to apply these recent theoretical advances in non-normal time-dependent stability analysis to the forecast error problem. We intend to use methods of modern control theory, specifically balanced Hankel operator truncation methods, to reduce the dimension of the error system. Having reduced the order of the error system we will then obtain an effective Kalman gain for state identification in the full forecast system.

WORK COMPLETED

We have completed the fundamental theory of error growth in time dependent forecast systems as described by Lyapunov vector dynamics.

We have adapted the control theory approach of optimal balanced Hankel operator model order reduction previously used in association with discrete engineering systems to the time dependent forecast error dynamical system.

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We have constructed a reduced order Kalman filter using balanced truncation and demonstrated its ability to accurately observe the error in a Lyapunov unstable forecast trajectory.

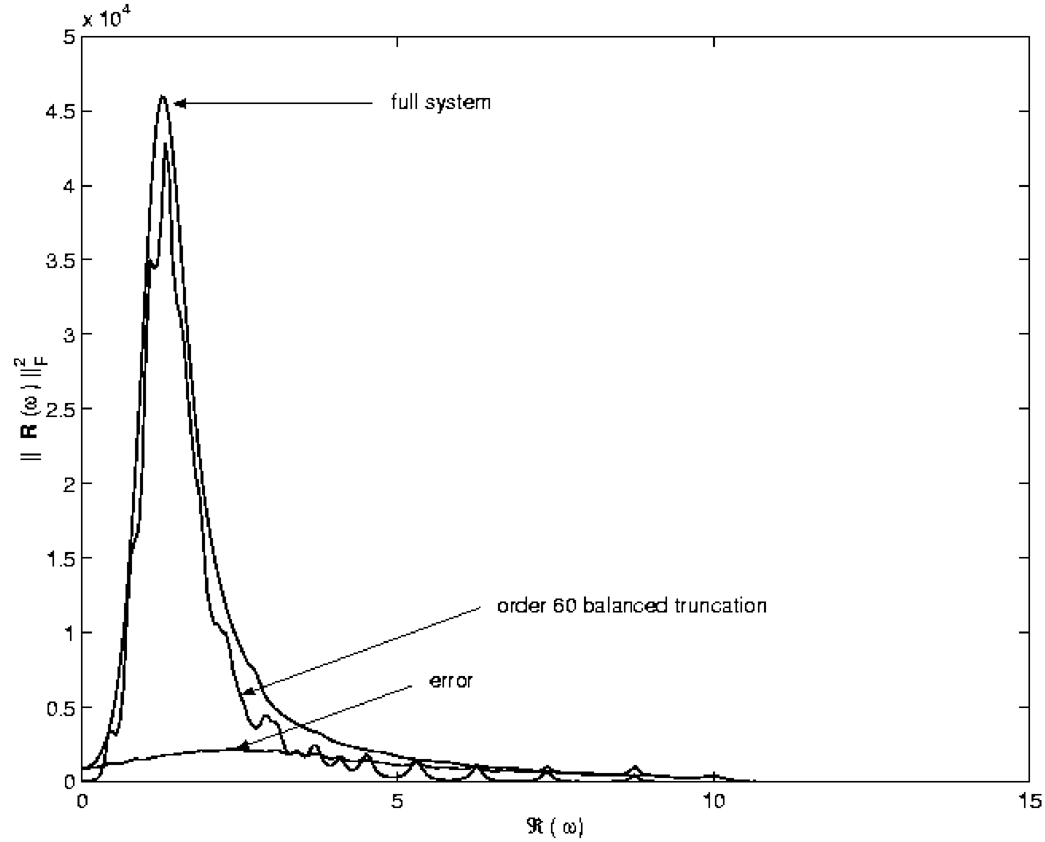

Fig. 1 : The variance of the full system, of the order 60 system obtained by balanced truncation, and the error incurred in the truncation as a function of frequency. The full system models error dynamics in an atmospheric midlatitude storm track with 400 degrees of freedom. The reduced system is seen to accurately approximate the full system over the frequency rangeof primary importance to forecast (0.5 1/d – 9 1/d) .

RESULTS

We have developed a theory for error structure in time dependent systems which facilitates understanding and prediction of error statistics and we have shown that the dominant error growth arises intrinsically from destabilization of a restricted set of non-normal vectors of the mean operator by time dependence. Together these results allow us to reduce the dimension of the unstable dynamics of time dependent error systems by representing the dynamics in a similarly restricted subspace.

We have developed a method for reducing the dimension of the time independent error system which retains the dominant non-normal subspace. This method of optimal model order reduction truncates the error dynamics in balanced coordinates in which the stochastic optimals and the empirical orthogonal functions collapse to the same set of structures (Moore, 1981; Glover, 1984; Farrell and Ioannou, 1996).

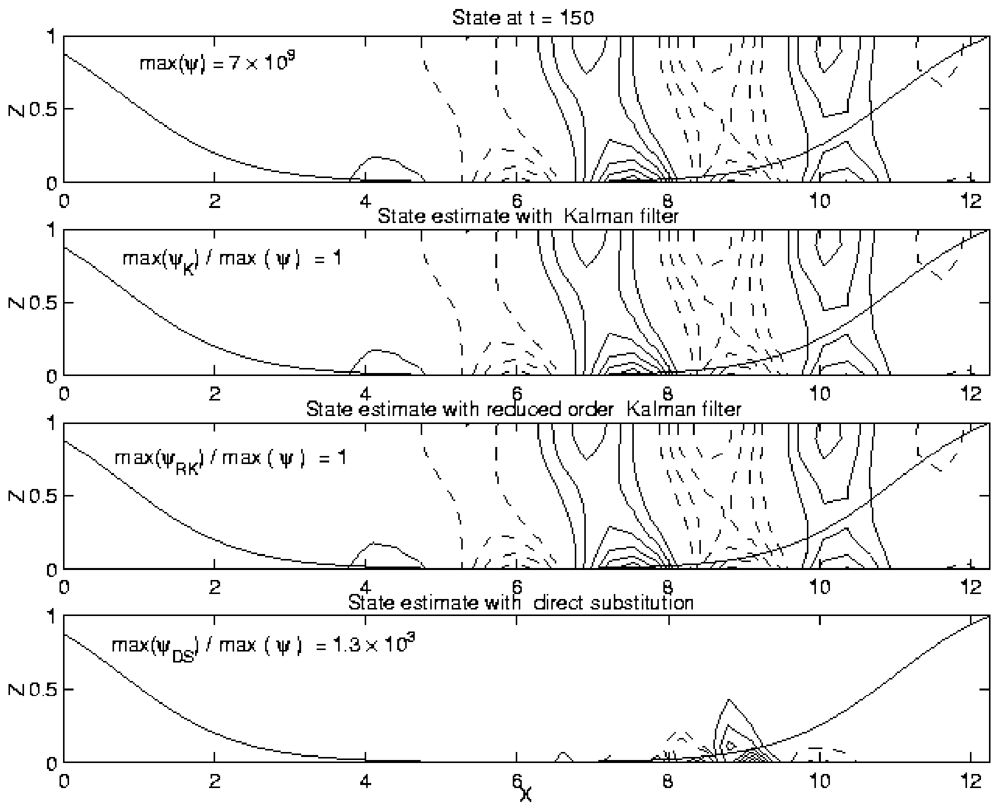

Fig. 2: First panel: streamfunction of the true state at t=150. The true state is Lyapunov unstable with Lyapunov exponent $\lambda \approx 0.075$. The true state corresponds to evolution of perturbations in the time varying flow of a midlatitude storm track. Observations of this true state are made at longitudes $x = 4.7$, $x = 6.9$ and at heights $z = 0.1$, $z = 0.78$, every 0.1 units of time. Second panel: streamfunction of the analyzed state as obtained using a full Kalman filter. Third panel: streamfunction of the analyzed state as obtained using a Kalman filter calculated from a balanced truncation of the full 400 degrees of freedom system to 40 degrees of freedom. Fourth panel: streamfunction obtained by direct substitution. The figure demonstrates that a Kalman filter obtained by reduction of the time dependent dynamics by an order of magnitude provides a near perfect identification of the Lyapunov unstable error structure of the time dependent flow. The effectiveness of the reduced order Kalman filter in reproducing the atmospheric state is contrasted with the failure of simple substitution to achieve this goal.

We have developed a method for quantifying the accuracy of the reduced order error system in the frequency domain (see Fig.1).We have tested the method of reducing the dimension of the forecast error systems in time independent error systems, as would be appropriate for forecast over a short enough interval that time dependence of the flow can be ignored (~24 hr in the atmosphere). In time independent error systems a very good approximation to the error dynamics can be obtained with dimension reduction by as much as a factor of 10 .

We have determined that truncating the dynamical system in a balanced realization of the optimals and evolved optimals for a single time provides a nearly optimal reduced order error system so long as the optimals are obtained for growth over an interval corresponding to that of the global optimal. This is a significant result because it suggests that implementing the order reduction algorithm in an operational forecast mode can be simplified computationally.

We have extended the balanced truncation model order reduction to time dependent Lyapunov unstable error systems in order to obtain a calculable reduced order Kalman filter appropriate for error observations in the time dependent tangent linear forecast system. We showed that this reduced order Kalman filter successfully observes a model of the atmospheric error state (see Fig. 2).

IMPACT/APPLICATION

The methods developed in this work can be directly extended to operational forecast models. Efficient calculation of error statistics using our results will allow implementation of advanced methods of state estimation such as the Kalman filter.

TRANSITIONS

None

RELATED PROJECTS

None

REFERENCES

Farrell, B. F. and Ioannou P. J. 1996: Generalized Stability. Part I: Autonomous Operators. Journal of the Atmospheric Sciences, 53, 2025-2041.

Glover, K. 1984: An optimal Hankel-norm approximation of linear multivariable systems and their L^∞ error bounds. International Journal of Control, 39, 1115-1193.

Moore, B. C., 1981: Principal component analysis in linear systems: controllability, observability, and model reduction. IEEE Trans. on Automatic Control, AC-26, 17-31.

PUBLICATIONS

Farrell, B. F. and Ioannou, P. J. 1999: Perturbation growth and structure in time dependent flows. Journal of the Atmospheric Sciences, 56, 3622-3639.

Farrell, B. F. and Ioannou, P. J. 2000: Accurate low dimensional approximation of the linear dynamics of fluid flow. Journal of the Atmospheric Sciences. Accepted.

Farrell, B. F. and Ioannou, P. J. 2000: State estimation using a reduced order Kalman filter. Journal of the Atmospheric Sciences. Submitted.